Kinematics & Dynamics of Linkages

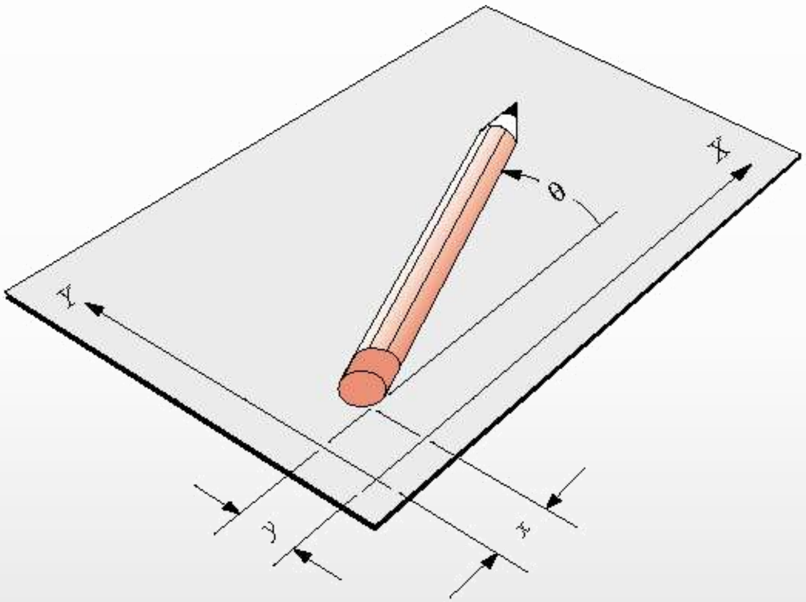
Lecture 2: Kinematics Fundamentals

Ali H. Ammouri

Spring 2018

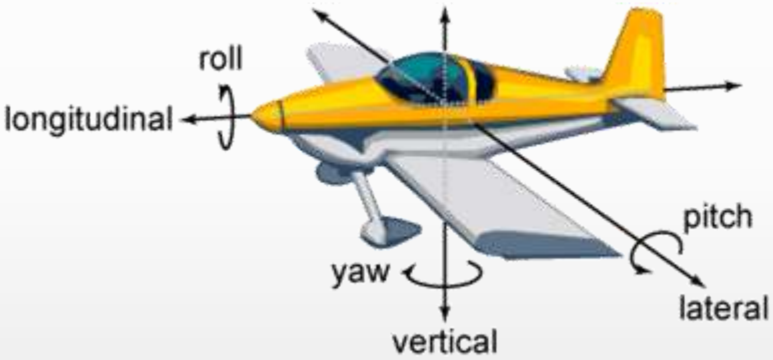
Mechanical Systems

- Classified by their Mobility (M) or degrees of freedom (DOF)
- DOF: the number of independent parameters needed to uniquely define their positions in space at any instant with respect to a selected frame of reference (the parameters are not unique)



Kinematic Principles

- A rigid body in a plane has 3 DOF
  - 2 lengths & 1 angle
- A rigid body in 3D space has 6 DOF
  - 3 lengths & 3 angles
- A rigid body = kinematic link
- Kinematic bodies are rigid & massless



https://qph.ec.quoracdn.net/main-qimg-36f3e430bfabca0fb825b89a33cd9f03

Types of Motion

- Translation
  - The linear position of a body changes with respect to a fixed frame
- Rotation
  - The angular orientation of a body changes about a fixed frame of reference
- Complex (General Motion)
  - Simultaneous combination of translation and rotation motions

An example of rotation. Both the worm and the worm gear are rotating on their own axis.

https://us.framo-morat.com/wp-content/uploads/sites/8/Schneck_Katalog.jpg

Linkages

- Basic building blocks for all mechanisms that are made up of **links** and **joints**
- **Link** = a rigid body possessing at least 2 nodes
- **Nodes** = points for attachment to other links

https://us.elkasuspension.com/wp-content/uploads/sites/5/2016/02/ATV-Linkage_2.jpg

Types of Links

Binary

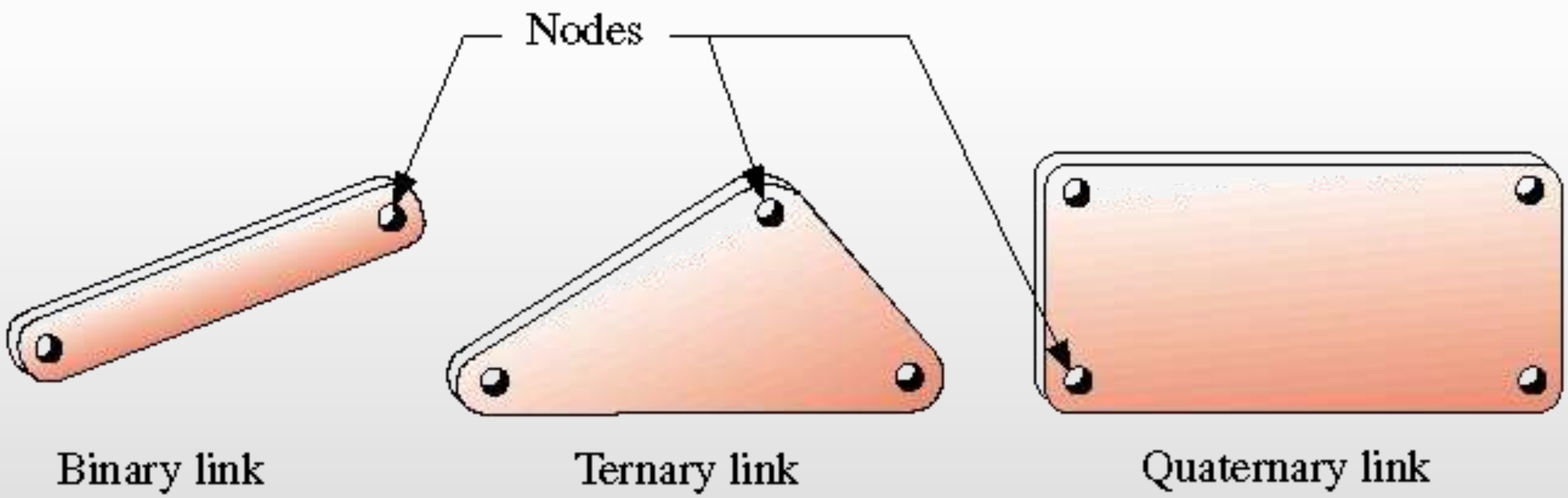
A link with 2 nodes

Ternary

A link with 3 nodes

Quaternary

A link with 4 nodes



Joints

- Connections between 2 or more links at their nodes
- Allow for a constrained motion between the connected links
- Also called "kinematic pairs"
- Classified in different ways

Joint Classification

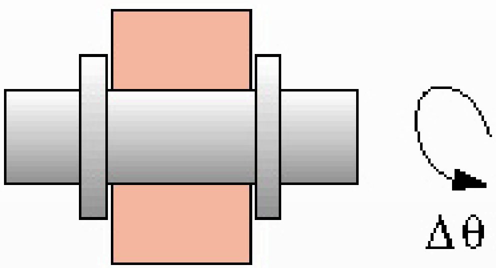
1. by the number of DOF allowed at the joint

2. by the type of contact between the elements
   - point, line or surface

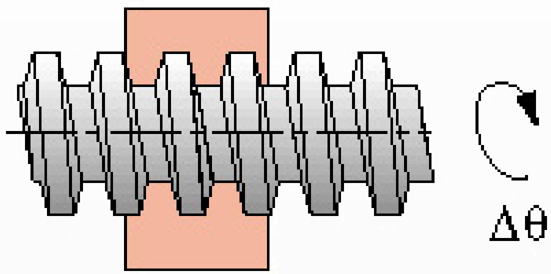
3. by the type of physical closure of the joint
   - force or form closed

4. by the number of links joined at the joint
   - order of the joint

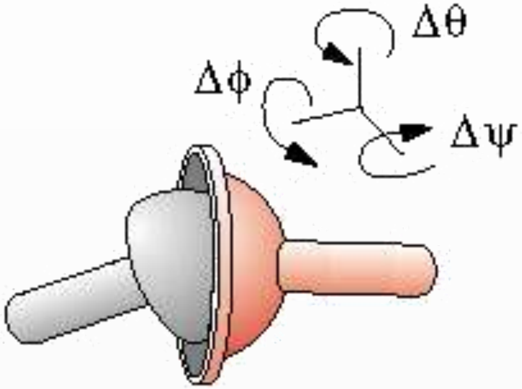
Classification of Joints by their DOF 1/2



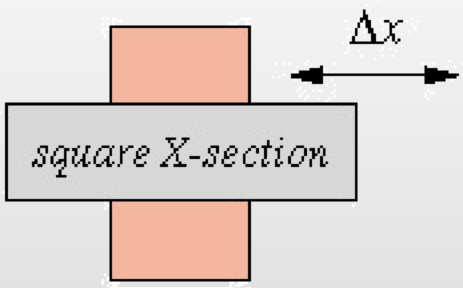
Revolute (R) joint—1 *DOF*



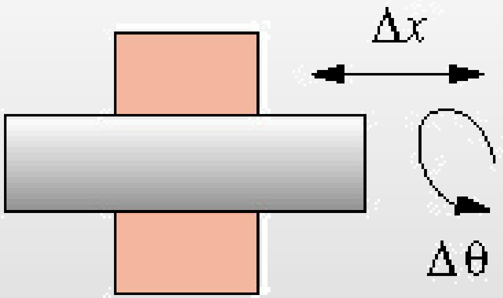
Helical (H) joint—1 *DOF*



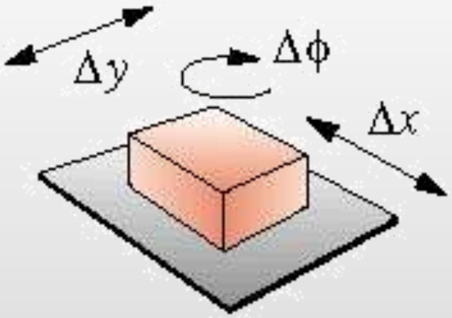
Spherical (S) joint—3 *DOF*



Prismatic (P) joint—1 *DOF*



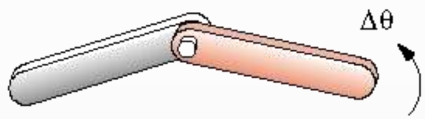
Cylindric (C) joint—2 *DOF*



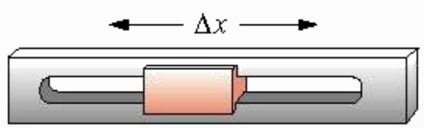
Planar (F) joint—3 *DOF*

Classification of Joints by their DOF 2/2

- Full Joint: Rotating pin or translating slider (1 DOF)

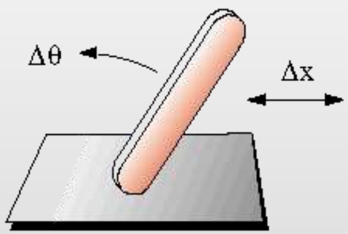


Rotating full pin (R) joint (form closed)

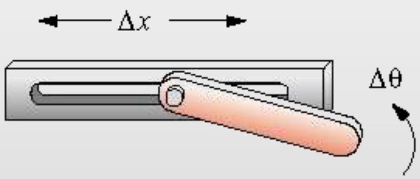


Translating full slider (P) joint (form closed)

- Half Joint: Roll-slide joint (2 DOF)



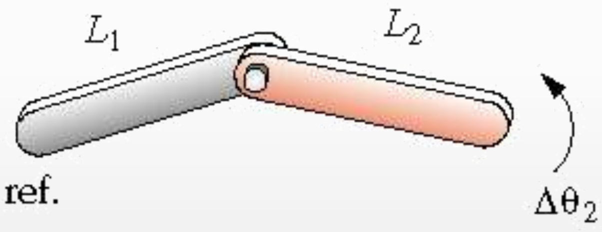
Link against plane (force closed)



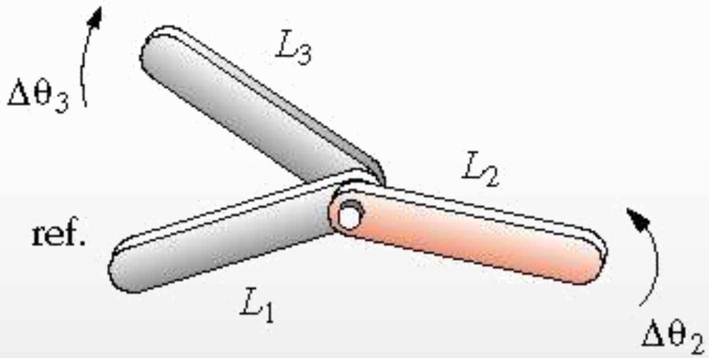
Pin in slot (form closed)

Order of a full joint

- One less than the number of links joined



First order pin joint - one *DOF*
(two links joined)



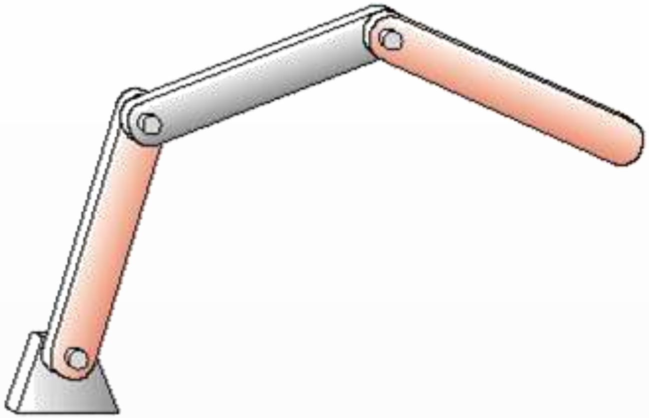
Second order pin joint - two *DOF*
(three links joined)

Definitions

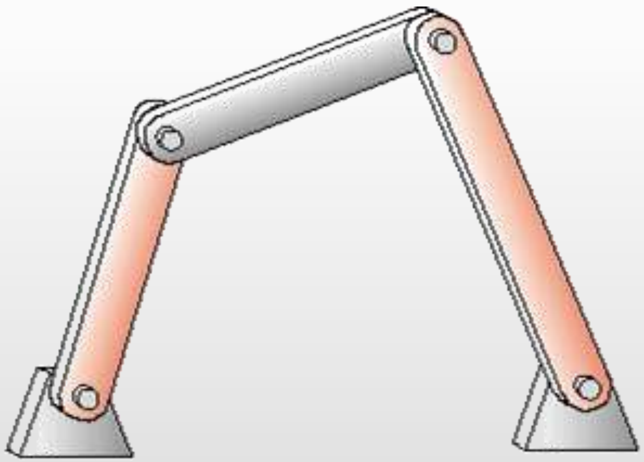
- Kinematic Chain
  - An assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied motion
- Mechanism
  - A kinematic chain in which at least one link has been grounded, or attached to the frame of reference
- Machine
  - A collection of mechanisms arranged to transmit forces and do work

Determining Mobility

- Need to know:
  - # of links
  - # of joints
  - Interaction among them
- **Closed** mechanism chain
  - M = 1 or less DOF
- **Open** mechanism chain
  - M = More than 1 DOF



(a) Open mechanism chain



(b) Closed mechanism chain

Gruebler Condition

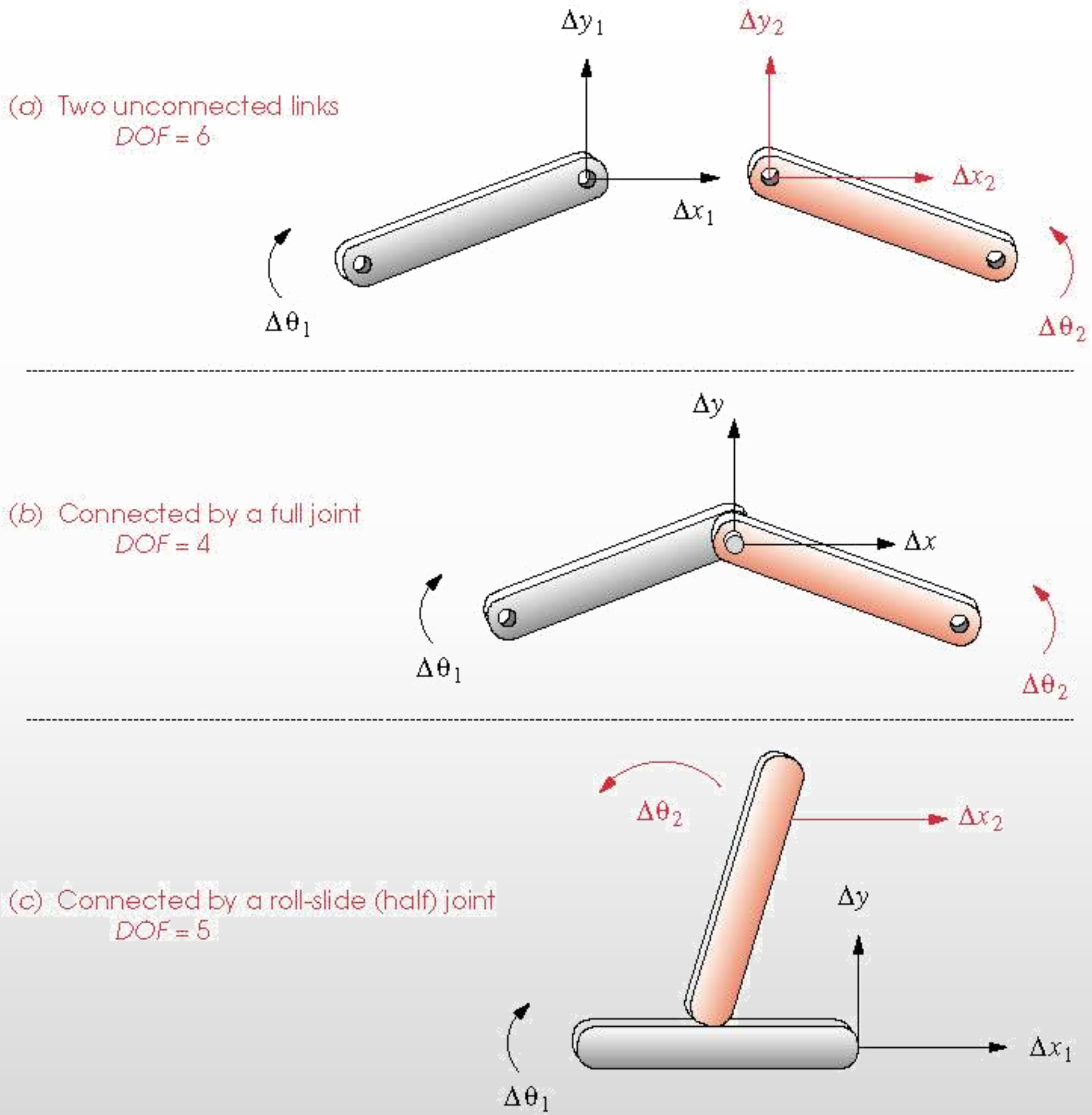
- Any link in a plane has 3 DOF
    - therefore, a system of L unconnected links in the same plane will have 3L DOF
- When 2 links are connected by a full joint
    - 2 DOF will be removed (constrained)
- When 2 links are connected by a half joint
    - 1 DOF will be removed (constrained)
- When a link is grounded (attached to the reference plane)
    - 3 DOF will be removed (constrained)

Example

a) 2L DOF

b) 2L-2 DOF

c) 2L-1 DOF



Gruebler's Equation

- $M = 3L - 2J - 3G$
  - Where:
    - L = # of links
    - J = # of joints
    - G = # of grounded links
- In a real mechanism, even if more than 1 link is grounded, the net effect will be to create one larger ground link, as there is only one ground plane
  - Therefore,
    - $G = 1$
- Gruebler's equation becomes:
    - $M = 3(L - 1) - 2J$

Kutzbach's Equation

- Takes into account the value of all joints
  - Full and half

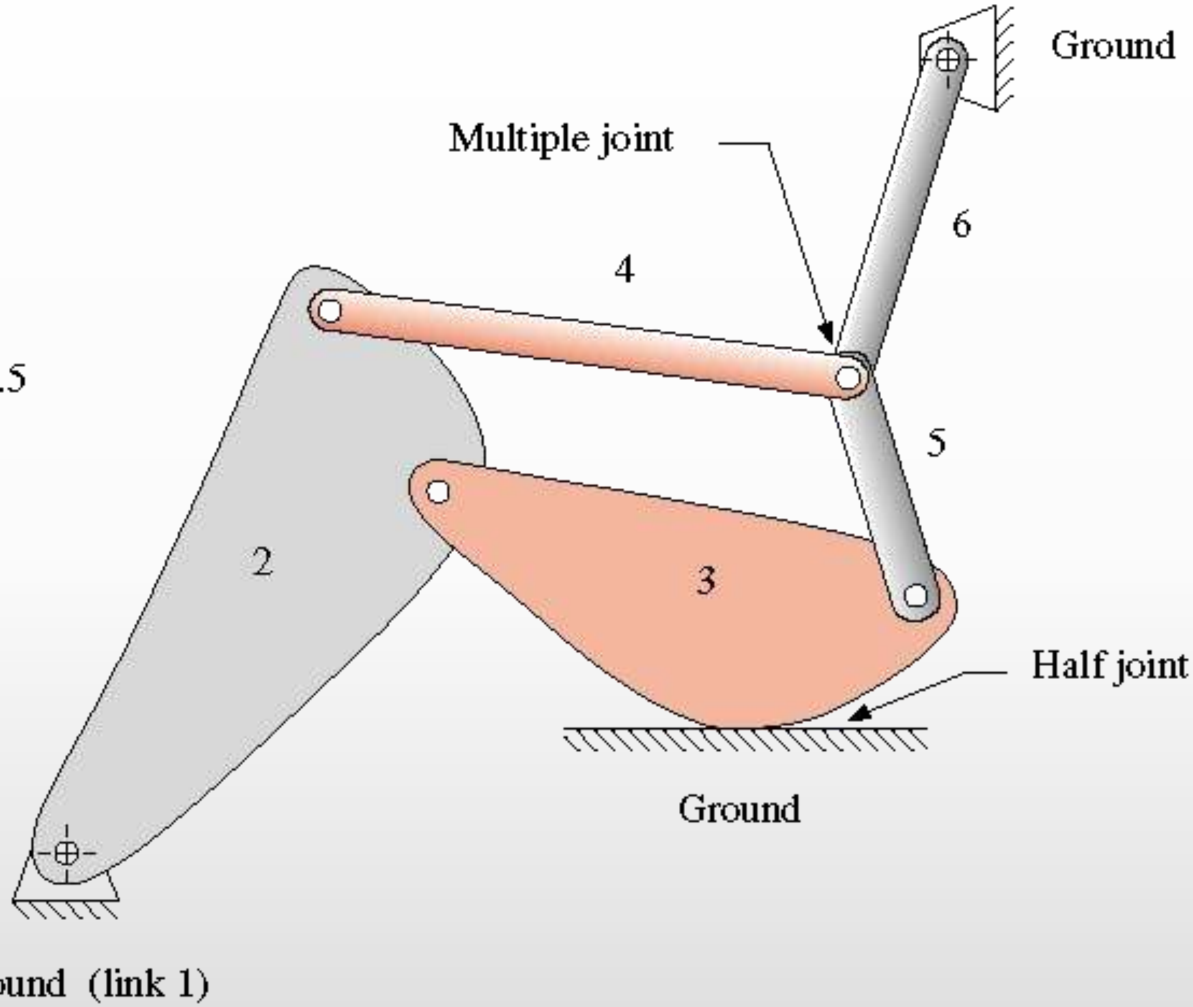
- $M = 3(L - 1) - 2J1 - J2$
    - Where:
      - L = # of links
      - J1 = # of full joints
      - J2 = # of half joints

Example

M=3(L-1)-2J

$L = 6, \quad J = 7.5$

$DOF = 0$



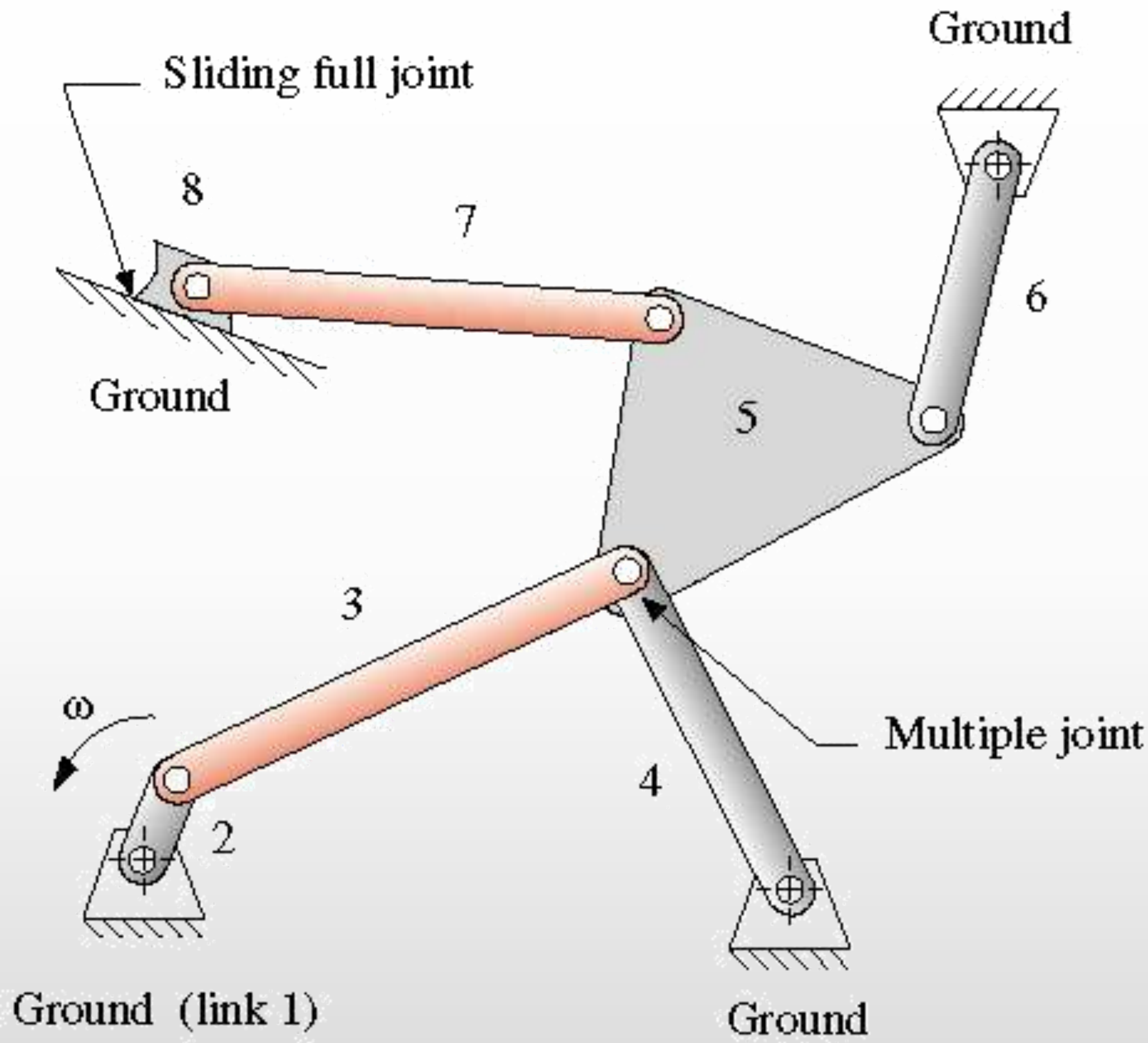
(b) Linkage with full, half, and multiple joints

Example

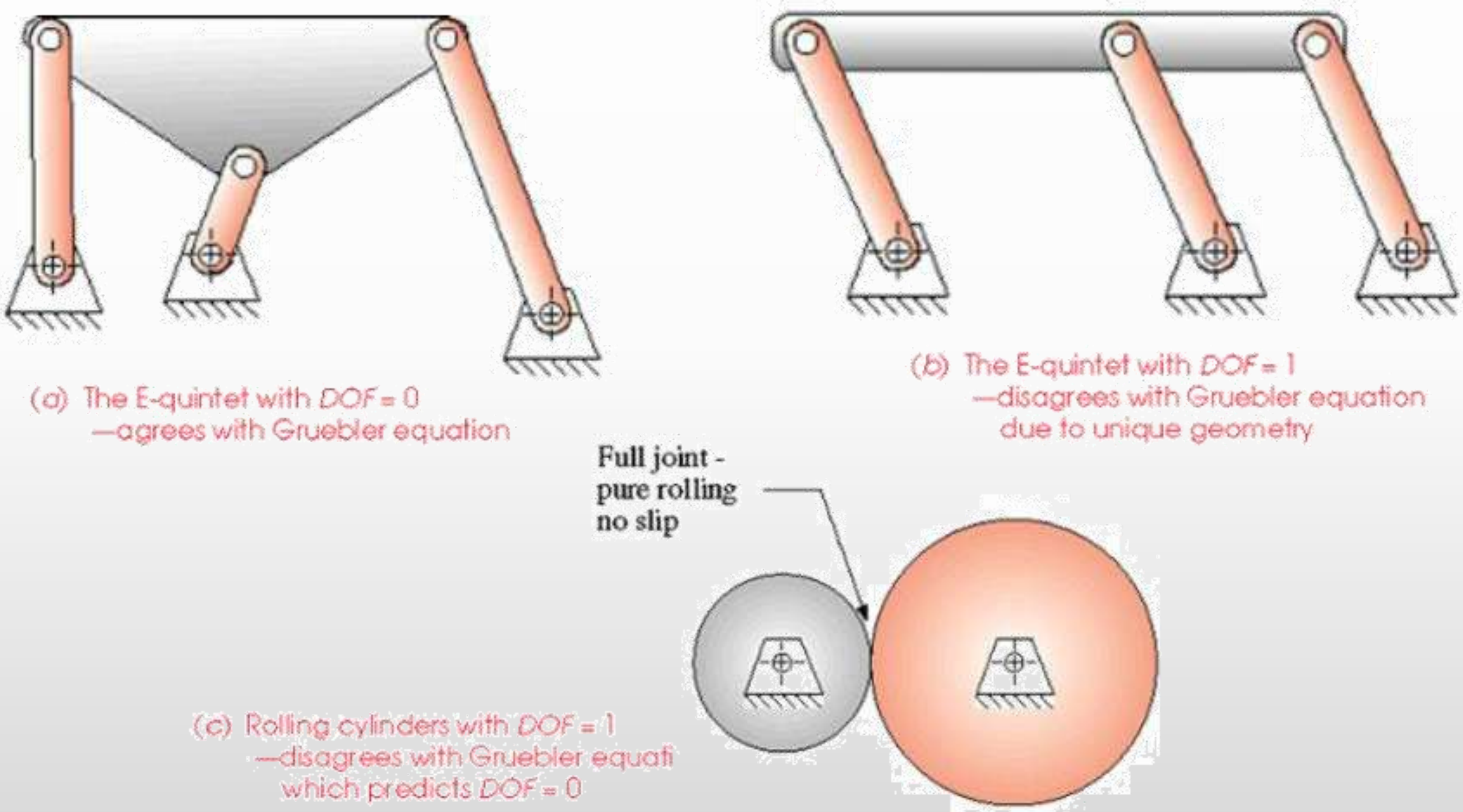
M=3(L-1)-2J

$L = 8, \quad J = 10$

$DOF = 1$



Paradoxes to Gruebler's Equation



(a) The E-quintet with $DOF = 0$
—agrees with Gruebler equation

(b) The E-quintet with $DOF = 1$
—disagrees with Gruebler equation
due to unique geometry

(c) Rolling cylinders with $DOF = 1$
—disagrees with Gruebler equati
which predicts $DOF = 0$